
Aerodynamic Characteristics of Arbitrary Three-Dimensional Shapes at Hypersonic Speeds

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ABSTRACT

Techniques for aerodynamic analysis of complex shapes in hypersonic flow are presented. The analysis is capable of handling completely arbitrary shapes and provides optional force-calculation methods for use as required. Techniques for describing arbitrary shapes are presented and the various force-calculation methods and their applicability are discussed. A procedure developed to check the accuracy of the geometric data uses a computer and automatic recorder to draw pictures of the vehicle viewed from any angle. Several examples demonstrate the use of this analysis in design studies. The calculated results are in excellent agreement with experimental data, even for complex shapes previously difficult to analyse.

1. INTRODUCTION

When the hypersonic aerodynamicist starts to make an analysis of a vehicle, he is faced with two basic problems. First, the vehicle shape may be so complex that it is difficult to describe with any analytical expression. Second, the best method for calculating the forces on the vehicle depends on the vehicle shape and the flight condition. The geometric description and the force-calculation procedure are frequently fused together in a single analytical expression for convenient solution. As an alternative approach, the aerospace designer frequently restricts his choice to those shapes that are amenable to theoretical calculations. However, to obtain optimum designs for such vehicles as the aerospace-plane and advanced lifting entry vehicles, the designer cannot be restricted to the use of combinations of simple shapes, but must be free to shape the vehicle as necessary to meet the design requirements. In the pre-

liminary design phase, the designer must investigate a large number of vehicle shapes before resorting to wind-tunnel testing.

Such problems present a difficult task for the aerodynamicist who must estimate the vehicle aerodynamic characteristics. The work described in this paper is focused on the derivation of an analysis system capable of solving these problems. This paper presents a brief description of the system in its present stage of development and gives several examples to illustrate its use. The numerical calculations are performed on a large-scale digital computer. Important components of the computer program system are reviewed briefly. Emphasis in this paper is on the development of the analysis system itself, not on the details of the theoretical methods involved. This paper is a summary of the work described in Ref. 1.

SYMBOLS

C	form of Chapman-Rubesin viscosity coefficient $(\mu_w/\mu_\infty)(T_\infty/T_w)$
C_D	drag coefficient
C_L	lift coefficient
C_m	pitching-moment coefficient
C_N	normal-force coefficient
C_p	local surface-pressure coefficient
K	modified Newtonian correlation factor (= 2 for Newtonian)
L/D	lift-drag ratio
M	Mach number
Re_x	Reynolds number
S	boundary-curve length
T	temperature at wall $(\)_w$ and in free stream $(\)_\infty$
u, w	position co-ordinates on parametric surface
X, Y, Z	co-ordinates of a surface point
\bar{V}_∞	hypersonic viscous parameter = $M\sqrt{C/Re_x}$
α	angle of attack
δ	angle between surface and free stream
μ	gas viscosity at wall $(\)_w$ and in free stream $(\)_\infty$
ϕ	lower-flat-surface angle for slab delta

2. ANALYSIS TECHNIQUES

Background and analysis requirements

The various approaches used in calculating aerodynamic forces on three-dimensional shapes differ in the methods used to attack the two basic problems — the problem of geometric representation and the problem of

selecting the force-calculation method. Most of the literature is concerned with the development of solutions for the forces on given simple shapes. Frequently, shapes are selected for which exact results may be obtained. Such approaches have been used because they offer the best way of gaining a basic understanding of these very complex problems.

Several publications give excellent reviews of the present status of knowledge in hypersonic aerodynamics^(2, 3, 4). The task remains, however, for the aerodynamicist to develop techniques for applying this knowledge to problems encountered in design studies for tomorrow's hypersonic vehicles. The primary purpose of the work outlined in this paper is to develop an analysis system capable of describing arbitrary three-dimensional shapes and containing a number of different force-calculation methods. The term 'analysis system' is used because several computer-program components are required to meet the desired objectives. The components fall into three categories: (i) Geometric Data Preparation, (ii) Aerodynamic Analysis, and (iii) Output Data Presentation. With the availability of the proper input-output equipment and a large digital computer, these components can be operated as an integrated system.

Geometric description

The first (and certainly the basic) problem is to select a method for representing the vehicle shape. For maximum utility, the method selected should be applied in a manner that permits the description of a vehicle on a component build-up basis. This gives increased flexibility in shape description and makes it possible to use different force-calculation methods for different components. Because of possible changes in the surface contours of a component, it is necessary to divide the component into several sections. The discussions that follow are concerned with the methods used to describe the shape of a given section of a vehicle component.

Surface description methods fall into two broad categories: (i) distribution of a very large number of points in space organised to form small plane surfaces and (ii) use of surface-fit techniques to derive a mathematical equation that approximates the shape of the section. There are several kinds of approaches in the use of each method, but only two will be discussed. These are identified as the 'distributed-plane-quadrilateral method' and the 'mathematical surface-fit technique.' Both of these methods are in use at Douglas. Each method has its advantages and disadvantages. The distributed-plane-element method was selected for use in most of the examples shown in this paper because of its simplicity and because of its usefulness in illustrating certain important features of an arbitrary-body analysis system. Since it was also the earliest method used at Douglas, it is further developed and has been used on a large number of practical design studies.

The distributed-element method has two basic characteristics — one is simplicity, and the other is the large number of points required to describe a complex shape. The distributed-plane-element method as used in this paper was developed at Douglas by J. L. Hess and A. M. O. Smith for use in three-dimensional potential-flow problems⁽⁵⁾. Since a complete derivation of the method is available,^(1, 5) only a brief summary is presented below.

Each section of a vehicle component is further divided into a number of small units called elements, each defined by four points in space. If the four related points of each element are connected by straight lines we have a picture that shows how the elements are used to describe a given section (Fig. 1). In practice, the surface co-ordinates are usually recorded from cross-

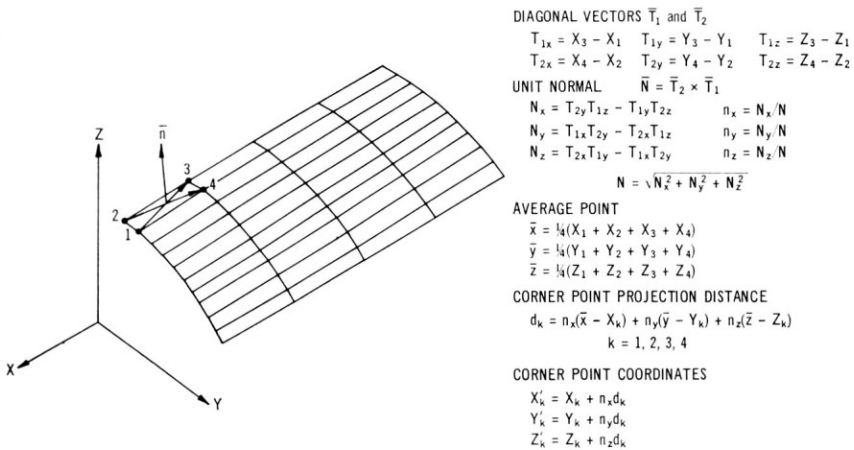


FIG. 1 — Surface representation by plane-quadrilateral method

section drawings of the vehicle in such a way that each point need be read only once (even though it may be a member of as many as four adjacent elements). Each point is defined by its three co-ordinates and a status parameter that indicates whether it is the first point of a new section, a continuation of a column of points, the beginning of a new column, or the last point of the vehicle.

Each set of four points is converted into a plane-quadrilateral element by the procedure outlined in Fig. 1. The normal to the quadrilateral is taken as the cross product of two diagonal vectors formed between opposite element points. The order of the input points and the manner of defining the diagonal vectors is used to ensure that the cross product gives an outward normal to the body surface. The next step is to define the plane of the element by determining the averages of the co-ordinates of the original four corner points. These points are then projected parallel to the normal vector into the plane

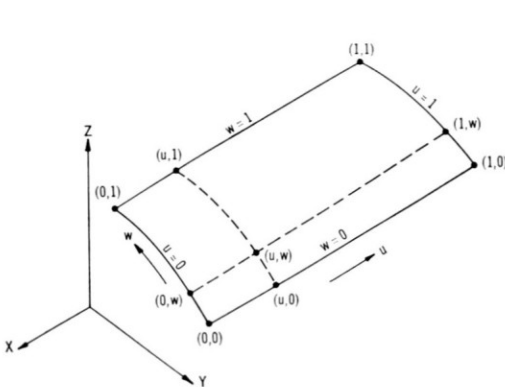
of the element to give the corners of the plane quadrilateral. The corner points of the quadrilateral are equidistant from the four points used to form the element. Additional parameters required for subsequent force calculations, quadrilateral area and centroid, may now be calculated.

The spacing and orientation of the elements is varied in such a way that they describe the vehicle shape accurately. Since four points are used to define the plane quadrilateral, the edges of adjacent elements are not coincident. This is not important, since the pressure is calculated only at the quadrilateral centroid. This pressure is then assumed to be constant over the surface of the element.

The plane-quadrilateral surface description method is not as elaborate as some of the other methods. It is important, however, to note that the simplicity of the method permits the use of conventional cross-sectional drawings in data preparation (no surface slopes required) and the use of semi-automatic data-reading techniques. Also, as subsequent illustrations will demonstrate, computer-generated pictures are used in checking the geometric data for errors.

Several different mathematical surface-fit techniques are described in the literature. The one described below, which was adapted from the formulation given by Coons⁽⁶⁾, also divides the vehicle component into a number of sections or patches. The size and location of a section depends upon the shape of the surface.

The same section used in the discussion of the plane-distributed element method is depicted in Fig. 2 with the terminology of the mathematical surface-fit technique. The X, Y, Z co-ordinates of a point on the surface are related to the two parametric variables u and w . Thus, a surface in space is mapped into the u, w unit square. The basic problem is to find the position (X, Y, Z) of a point (u, w) in the interior of the section surface. The general procedure



BOUNDARY CURVE (FOR $u = 0$)

$$X_i(0, w) = Aw^3 + Bw^2 + Cw + D$$

$$A = 2[X_i(0,0) - X_i(0,1)] + \frac{\partial X_i}{\partial w}(0,0) + \frac{\partial X_i}{\partial w}(0,1)$$

$$B = 3[X_i(0,1) - X_i(0,0)] - 2\frac{\partial X_i}{\partial w}(0,0) - \frac{\partial X_i}{\partial w}(0,1)$$

$$C = \frac{\partial X_i}{\partial w}(0,0) \quad D = X_i(0,0)$$

$$\frac{\partial X_i}{\partial w} = \frac{\partial X_i}{\partial S} \frac{\partial S}{\partial w} \quad i = 1, 2, 3 \text{ FOR } X, Y, Z$$

BLENDING FUNCTIONS

$$F_1(u) = 3u^2 - 2u^3 \quad F_2(w) = 3w^2 - 2w^3$$

$$F_0(u) = 1 - F_1(u) \quad F_0(w) = 1 - F_2(w)$$

SURFACE FORM

$$X_i(u, w) = X_i(0, w)F_0(u) + X_i(1, w)F_1(u) + X_i(u, 0)F_0(w) + X_i(u, 1)F_1(w) - X_i(0, 0)F_0(u)F_0(w) - X_i(0, 1)F_0(u)F_1(w) - X_i(1, 0)F_1(u)F_0(w) - X_i(1, 1)F_1(u)F_1(w)$$

FIG. 2 — Surface representation by mathematical surface-fit method

is to first find relationships for the four boundary curves. These are defined as third-order polynomials in terms of the parametric variables. The points on the boundary curves corresponding to u and w (O, w and u, O , etc.) are then calculated. A general surface equation is used to calculate the properties at the point u, w . This equation uses blending or weighting functions to introduce properly the influence of each of the related boundary-curve points and the four corner points. The blending functions also ensure the continuity of the slopes across the boundaries between adjacent sections.

There are several methods for calculating the direction cosines of the tangent vectors required in the calculation of the corner-point derivatives. Most require the specification of additional surface-boundary points, some of which may lie on the extensions of the boundary curves. The derivatives must be calculated, since it would not be practical to measure them directly from drawings. One method is to pass circular arcs through three boundary curve points, the middle one being a corner point. The terms $\partial S/\partial w$ may be evaluated directly at the corner point by using the data required by the tangent vector calculations, or, with the additional boundary-curve points specified, $\partial S/\partial w$ may be replaced by $\Delta S/\Delta w$, where ΔS is the total length of the boundary and $\Delta w=1$. Once the surface has been described, it is a relatively simple matter to calculate the required normals to the surface.

One advantage of the mathematical surface-fit technique over the plane-distributed-element method is the smaller number of surface points required to describe a shape. However, additional points are required on the boundaries to determine the required corner derivatives. This method is not as adaptable to semi-automatic data-reading techniques, since the organisation of the required input data is more complex. The accuracy of this method depends upon the distribution and orientation of the surface sections, just as the plane-distributed-element method depends upon the distribution of the elements. An important advantage of the mathematical surface-fit technique is that surface derivatives can be easily determined. The surface-fit technique has also been used to generate parametric families of simple arbitrary shapes which were then converted into plane-distributed elements for subsequent calculations.

From the preceding discussion it may be seen that the final selection of the geometric technique for a given application depends upon the detail requirements of each system. Regardless of the method used, a difficult problem exists in that a large amount of data must be transformed from vehicle drawings into a form suitable for the computer. This may be accomplished by reading and recording the co-ordinates manually or by the use of semi-automatic $X-Y$ read-out equipment. But whatever process is used, it is subject to human error, and unless these errors can be easily detected and corrected, any method of describing arbitrary shapes is impractical. Fortunately, with modern graphical-display equipment and computers this problem

may be easily solved. The process is best illustrated by the example shown in Fig. 3. This picture of a high- L/D re-entry vehicle design was drawn by a computer from the input geometric data. The data errors are obvious and can

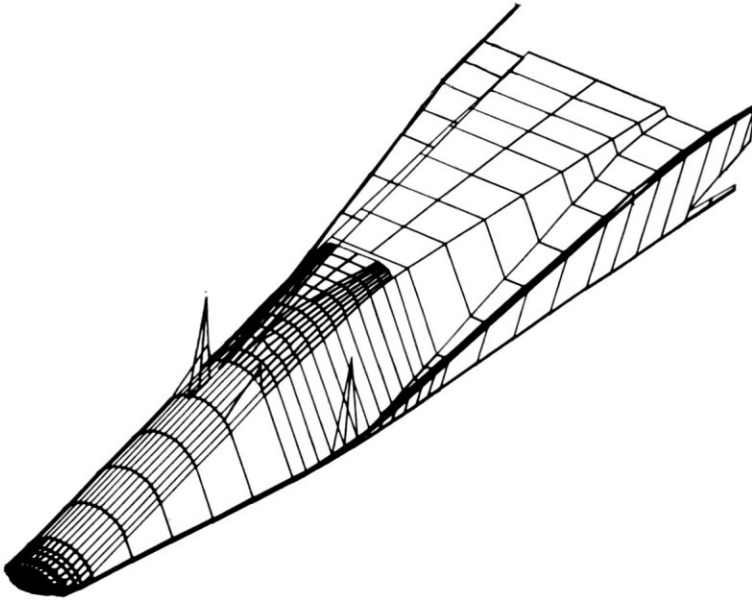


FIG. 3 — Error detection in geometric representation by use of computer-drawn pictures (high- L/D re-entry vehicle)

be quickly corrected. The geometric representation for a hypersonic cruise-vehicle wind-tunnel model is shown in Fig. 4.

The pictures shown in this paper are generated by a transformation of the vehicle co-ordinates with the required rotation matrices to give the desired viewing angle. A graphical display computer then draws straight lines between the corner points of each element on a special cathode ray tube. The image is then photographed. All the pictures shown use the distributed-plane-element method. The unit normal for each quadrilateral is also transformed with the rotation matrices and the resulting component out of the plane of the picture calculated. If this component is positive, the element faces the viewer and therefore is drawn by the computer. If the component is negative, the element faces away from the viewer and the element is not drawn. This procedure yields very realistic drawings of the vehicle from any desired viewing angle and serves both to check the geometry data for errors and to illustrate the method used in describing the shape for the computer. Most of the pictures shown were generated with a Stromberg-Carlson SC-4020 Data Recorder.

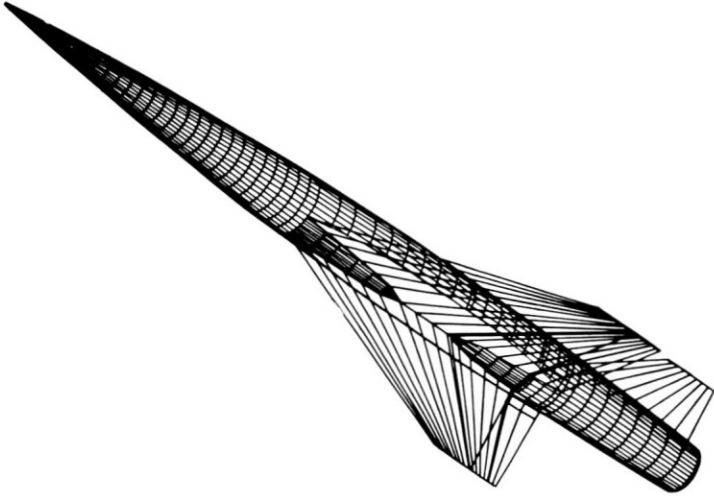


FIG. 4 — Geometric representation of hypersonic cruise vehicle

3. FORCE CALCULATIONS

Inviscid pressures

The aerodynamic literature contains descriptions of many different methods for calculating the pressures on hypersonic vehicles. Each method is tailored to a particular application, either by the geometry assumed or by the assumptions made in the gas-dynamics relationships. It is obvious that no one method will suffice for all shapes and flight conditions. Indeed, different methods must frequently be used for different components of a single vehicle. The logical conclusion is that the analysis system must include a number of different force-calculation methods to use for different vehicle component shapes and flight conditions. As new methods are devised and validated, they are added to the analysis system.

Many of the methods used in the analysis of high-speed shapes are listed in Fig. 5. An attempt has been made in the preparation of this figure to indicate the inter-relationships of the methods (the information can, of course, be organised in many different ways). Some of these methods are more applicable to the arbitrary-body problem than others.

The method of characteristics is the eventual ideal approach for the calculation of forces on three-dimensional shapes at high speeds. It will require starting solutions for three-dimensional blunt bodies of arbitrary shape. The development of a method of calculating three-dimensional boundary layers would permit the use of an iterative process to account for

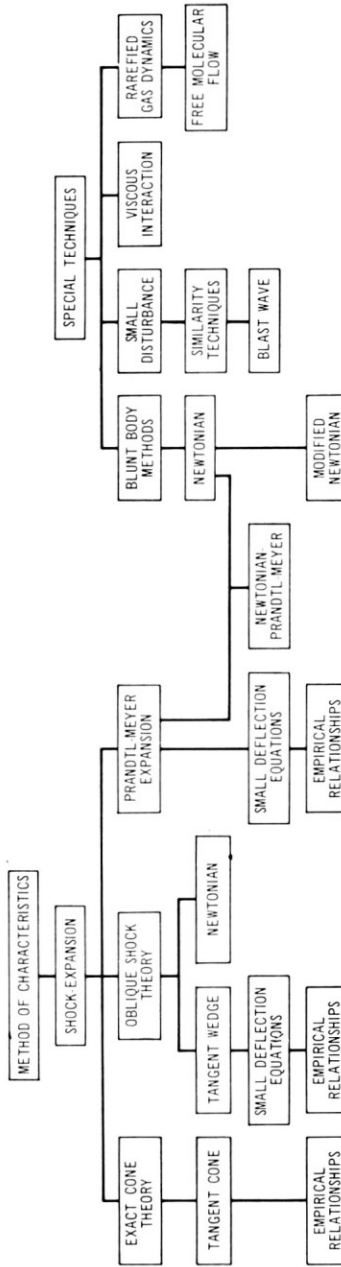


FIG. 5 — Inviscid force calculation methods

the viscous-inviscid interaction. Although this approach has been used for some very simple shapes⁽⁷⁾, the complete solution for arbitrary shapes is some time away. Significant progress is being made in the solution of the inviscid flow field by the method of characteristics^(8, 9). However, present mathematical techniques and digital-computer size and speed capability prevent application to typical preliminary-design problems. Application must be reserved for simple shapes or important detail design applications where very large computer times might be acceptable.

Many of the other methods shown in Fig. 5 would be useful force-calculation methods for inclusion in an arbitrary-body system. The selection of the proper method in a given application depends upon the vehicle-component shape and flight condition and must be selected by the engineer on the basis of his knowledge and experience in the use of each method.

The hypersonic-force-analysis system described in this paper contains the following force-calculation methods: shock expansion, tangent wedge, tangent wedge and tangent cone empirical⁽¹⁾, modified Newtonian, blunt-body empirical⁽¹⁰⁾, small-deflection oblique shock and expansion relationships⁽³⁾, several built-in modified Newtonian relationships^(11, 12), free-molecular flow⁽⁴⁾, Newtonian-Prandtl-Meyer blunt body⁽¹³⁾, and input pressure coefficient. The method to be used in impact and shadow regions may be specified independently. In all methods except the shock expansion and the Newtonian-Prandtl-Meyer, the pressure depends only on the angle that the local surface makes with the free-stream flow. All the local slope-dependent methods are particularly applicable to the hypersonic arbitrary-body problem, since the interaction of different vehicle elements is assumed to be negligible. Fortunately, this is true of many hypersonic problems.

The shock-expansion and Newtonian-Prandtl-Meyer methods require that the surface streamlines be known. For two-dimensional surfaces strip-theory may be used, with the streamlines assumed to be in the free-stream direction. For axisymmetric bodies at small angles of attack, the streamlines may be assumed to lie along meridian planes. These methods are used only for these special conditions and are not used on completely arbitrary three-dimensional shapes.

The modified Newtonian method is probably the most widely used of all the hypersonic-force-analysis techniques because of its simplicity and experimentally confirmed accuracy for many hypersonic problems. In its purely empirical form, the modified Newtonian relationship is $C_p = K \sin^2 \delta$, where K is an empirical correlation factor. In general, K is a function of Mach number, angle of attack, and component shape. The use of relatively simple techniques such as modified Newtonian, when combined with empirical correlations and an arbitrary-body surface description method, provide a powerful tool for analysing many hypersonic shapes.

Modified Newtonian theory has been used as the basis for extensive

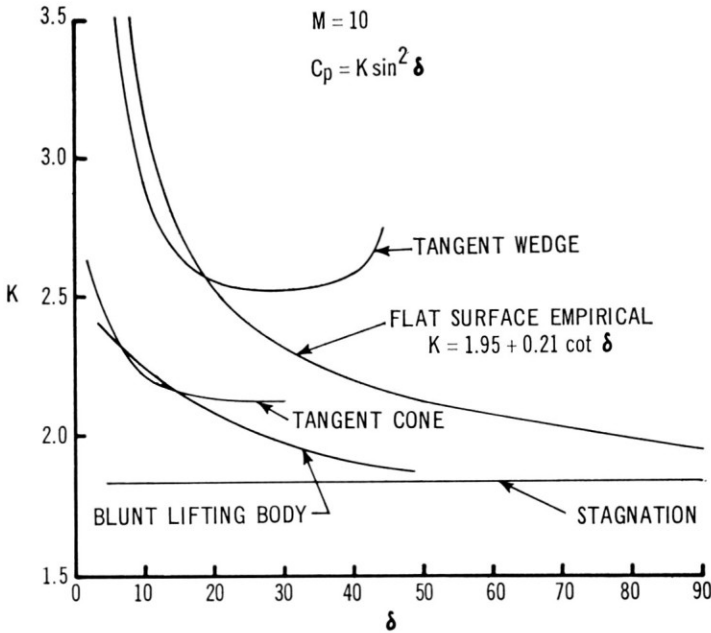


FIG. 6 — Modified Newtonian correlation factors

empirical correlations. Some of the fundamental ideas involved in the use of these techniques are illustrated in Fig. 6. For many problems the modified Newtonian correlation parameter, K , is held constant over the complete vehicle but is changed with angle of attack to reflect the general nature of the flow. This approach is identified as 'total-vehicle modified Newtonian', and is frequently used for calculations on very blunt shapes. The parameter K is found from empirical correlations of similar shapes. The blunt-lifting-body curve in Fig. 6 is an example of a correlation for a vehicle such as the NASA HL-10 re-entry body. In the pressure calculations for this case, δ is the local impact angle and K is a function of angle of attack. At high angles of attack, where the vehicle forces are dominated by a large blunt surface facing the flow, K approaches the stagnation value.

For vehicles that contain several different classes of component shape (such as blunt leading edges, flat lifting surfaces, hemispherical nose, etc.), it is necessary to use a different K for each component ('component modified Newtonian'). The stagnation K is used for blunt components, and higher values are used for the planar surfaces.

One further class of modified Newtonian is often required. The vehicle may have large curved areas that behave locally like flat surfaces. In this case K itself may be defined as a function of the local impact angle ('local modified Newtonian'). The flat-surface empirical relationship⁽¹²⁾ is an example of this

approach. In this case K approximates tangent-wedge results at low impact angles and approaches the stagnation K at high impact angles. Since the pressure is no longer a function of just $\sin^2 \delta$, this is not really a modified Newtonian method. However, because of its similarity, the modified Newtonian nomenclature is retained.

Viscous forces

The most difficult part of the analysis of arbitrary shapes is the calculation of the viscous forces. The natural complexity of the boundary-layer equations requires considerable simplification before solutions can be obtained. The analysis of the arbitrary-body boundary layer is hampered by these two basic problems: (i) The boundary-layer properties at a given point on the vehicle require knowledge of the previous flow history along the surface streamline; and, (ii) as a result of longitudinal and transverse surface curvature, complex pressure gradients are present. The problem of the three-dimensional boundary layer has been reviewed by Moore⁽¹⁴⁾ and will not be discussed in detail here. It is sufficient to state that a unified theory that deals with the three-dimensional boundary layer does not now exist. Solutions have been obtained only for particular problems^(15, 16, 17). Vaglio-Laurin⁽¹⁸⁾ presents a method for determining the surface streamlines from known surface-pressure distributions.

The problem remains, however, to adapt some procedure for use in the Hypersonic Arbitrary-Body Force Analysis System. Pending completion of more exact work, an engineering approach has been selected that retains the essential characteristics of the hypersonic-boundary-layer problem. In this approach, no attempt is made to calculate the detailed skin friction over the exact surface of the arbitrary shape. Instead, the total vehicle is represented by a number of flat surfaces so orientated as to approximate roughly the major features of the vehicle. The local inviscid flow properties on each of these surfaces is calculated by any of the methods provided for use in the detailed inviscid-pressure calculations. Once the local flow is known, the reference temperature method of Eckert⁽¹⁹⁾ is used to calculate the surface equilibrium temperature and the resulting laminar and turbulent viscous forces.

The interaction of the inviscid flow field and the boundary-layer growth has a pronounced effect on skin friction for flight conditions at high Mach number and low Reynolds number. In the hypersonic-force-analysis system, the method of White⁽²⁰⁾ is used to correct the laminar skin friction for this effect.

Integration of forces

Once the arbitrary-shape geometry is available and the pressures over the surface calculated, the integration of these pressures and conversion to con-

ventional aerodynamic forces becomes such a simple matter (*see* section 9–10 of Ref. 21) that only a few comments are necessary here.

In the analysis system described in this paper, the general expressions for the components of the free-stream velocity and local total-velocity vector are written in a general form and are not restricted either to small values of angle of attack, yaw angle, and roll angle or to small vehicle rolling, pitching, and yawing velocities. All the vehicle static-stability and damping derivatives are calculated by the method of small perturbations.

Special problems

In the development of an arbitrary-body analysis system such as that described here, several special problem areas should be mentioned. The most important of these is the possibility of interaction of the various vehicle components. The method of characteristics can give an exact evaluation of these complex three-dimensional effects. Lacking such detailed results, the use of approximate methods or interference factors will frequently give acceptable results. Certain types of configurations, such as the caret wings of Nonweiler⁽²²⁾ and the wave riders of Jones⁽²³⁾, must be analysed with special techniques. The shape of these configurations is selected in such a way that the exact aerodynamic characteristics may be easily calculated.

A special problem also occurs in the analysis of configurations with deflected control surfaces. At high Mach numbers, separated-flow regions will exist, causing large changes in control-surface effectiveness and in resulting vehicle characteristics. In the analysis system described, an empirical flow-separation criterion⁽²⁴⁾ has been used to assess these effects.

4. ANALYSIS SYSTEM

In the design of each component of this analysis system one objective was kept in mind — that the final system would be capable of operating as an integrated system, with the aerodynamicist communicating with the computer in a continuous man-machine interaction process. In an even broader concept, the complete aerodynamic analysis system would operate as a component of a much larger vehicle-design system involving other disciplines such as flight mechanics, aero-thermodynamic and structural mechanics. Such a design analysis system would permit the rapid generation of a vehicle concept, its evaluation in terms of flight performance and structural and weight analysis, and its re-cycling through a number of design trials to find the optimum vehicle to meet the design objectives. This larger system is beyond the scope of this paper.

The basic components of the Hypersonic Arbitrary-Body Force Analysis

System are illustrated in Fig. 7. The system operates at present without the continuous man-machine interaction process. This mode of operation must await the availability of the proper input-output facilities and associated computer equipment.

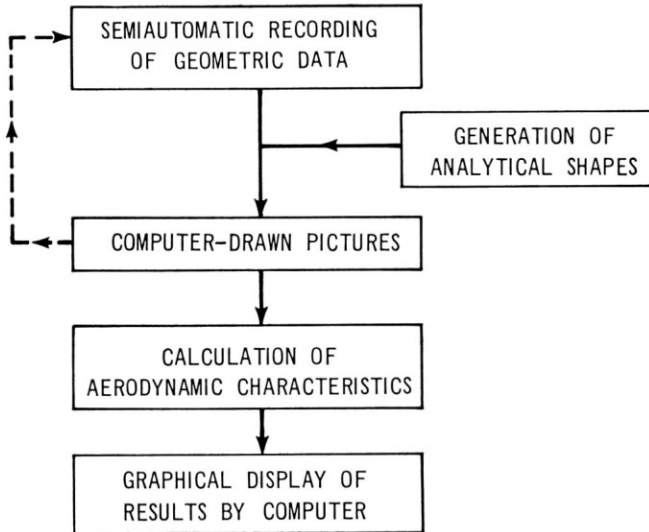


FIG. 7 — Hypersonic arbitrary-body analysis system components

The first three components of this system provide an accurate geometric description of the vehicle for use in the aerodynamic analysis. For a completely arbitrary shape, the geometric data are recorded from vehicle drawings either manually or by semi-automatic $X-Y$ reader-recorder equipment. For vehicles composed of simple analytical shapes, the detailed geometric data are calculated by a variety of optional analytical routines. The final step in the geometric preparation is to obtain computer-drawn pictures of the vehicle from various viewing angles to verify the data accuracy. This step has already been described. A rapid analysis of the geometric properties of a vehicle may also be made by using equipment such as that shown in Fig. 8.†

The major component of the analysis system is the Arbitrary Body Hypersonic Force Computer Program. This computer program is written on a modular basis and may quickly be modified to meet changing needs. Time-shared remote computer consoles are used to facilitate the checkout of these modifications. The final component of this system is the graphical display and recording of results by a computer.

† In the presentation of the paper a film was shown to illustrate this process.

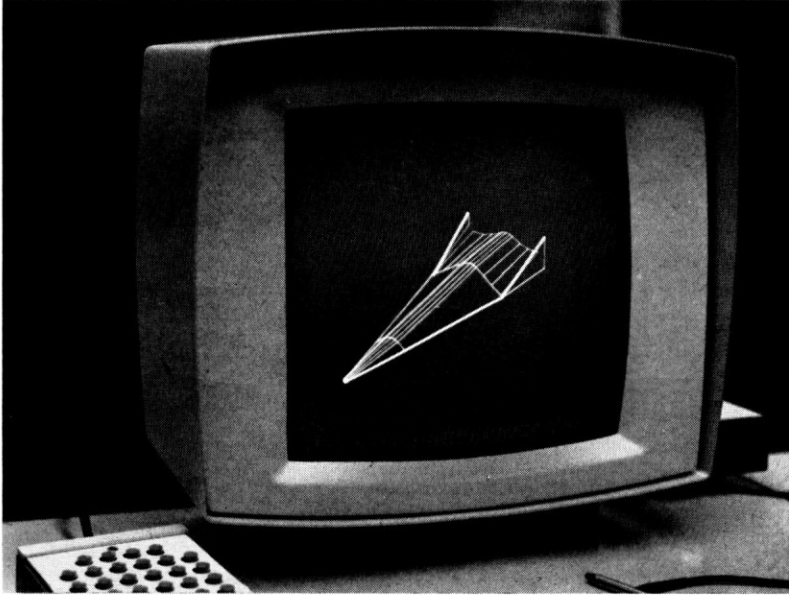


FIG. 8 — Rapid analysis of geometric properties by direct communication with computer (IBM 2250 display unit)

5. TYPICAL APPLICATIONS

The methods and analysis system discussed in this paper have been used to estimate the aerodynamic characteristics (both static and dynamic) of a number of vehicle designs. These include blunt entry bodies of medium lift-drag ratio, re-entry vehicles of high lift-drag ratio, and hypersonic aircraft of the cruise vehicle and the aerospace-plane type.

The first and most obvious application of an arbitrary-body analysis system is to that class of shape commonly known as the lifting-body re-entry vehicle. A classical example of this type of shape is the NASA HL-10 vehicle (Fig. 9). This configuration serves as an excellent example of the description of a shape by the plane-quadrilateral method and of the way the number of surface elements is changed to fit the surface contours. In each view in Fig. 9, the elements on the vehicle that face away from the viewer have been deleted automatically by the picture-drawing computer program, with the result that very realistic renditions of the vehicle are created. No convenient method has yet been devised for deleting those areas that face the viewer but are blocked by an intervening component of the vehicle (other than by manual retouching).

The HL-10 configuration provides an excellent example for the use of the

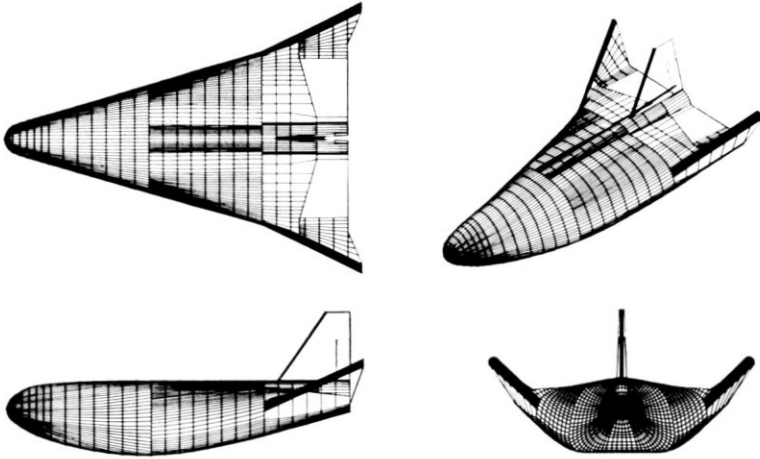


FIG. 9 — Geometric representation of NASA HL-10 lifting-body re-entry vehicle

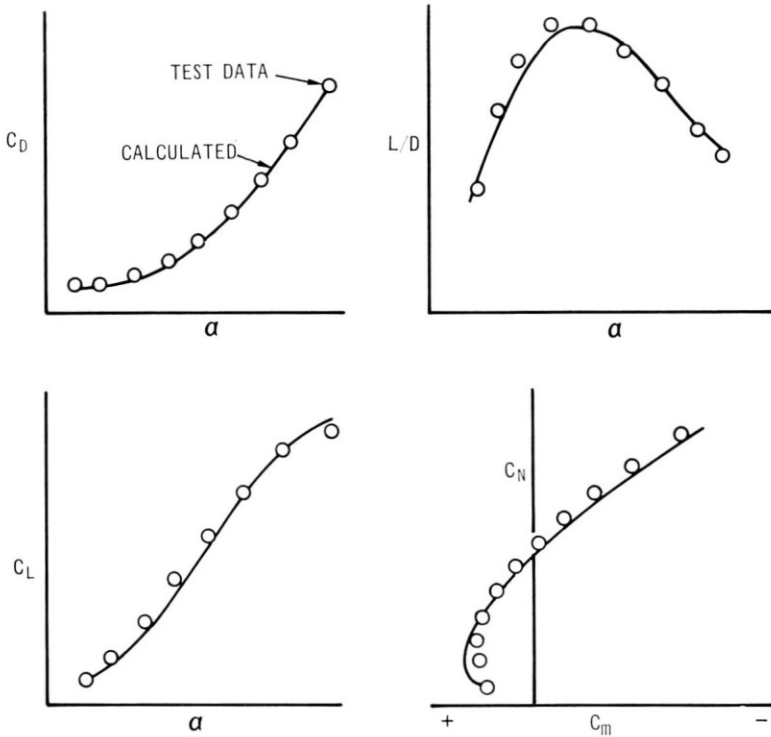


FIG. 10 — Comparison of calculated and wind-tunnel data for NASA HL-10 vehicle

modified Newtonian ideas expressed earlier in this paper. Typical results obtained from the computer program system are presented in Fig. 10. Wind-tunnel data are also presented on these plots to indicate the degree of agreement with the calculated values. As might be expected in the use of simple modified Newtonian theory, longitudinal pitching-moment characteristics do not agree as well as the lift and drag characteristics.

Another type of re-entry vehicle is shown in Fig. 11. This configuration was designed as the recoverable second stage of a two-stage horizontal take-off booster vehicle. The configuration was designed to have good volumetric efficiency and a high fineness ratio, with a flat lower lifting surface to obtain a high hypersonic lift-drag ratio. The low-speed and landing-flight phase was accomplished with the use of variable-sweep wings stored on the aft top of the vehicle during hypersonic flight. The pictures in Fig. 11 illustrate the use of computer-drawn pictures in visualising the geometric shape presented to the flow at various angles of attack.

The lift-drag-ratio characteristics of this configuration are shown in Fig. 12. These data illustrate the use of the arbitrary-body analysis system in determining the effects of vehicle size and flight condition on hypersonic performance. The hypersonic viscous parameter \bar{V}_∞ , has been shown⁽²⁵⁾ to be a useful parameter in correlating data for hypersonic vehicles.

In many design investigations it is desired to conduct studies of a large number of simple parametric shapes to gain an understanding of the effect of configuration variables on the vehicle aerodynamic characteristics. Such configurations can easily be derived by the analytical shape-generation techniques available in this analysis system. The objective here is to obtain the detailed geometric information required by the aerodynamic calculations with a minimum of input information. An example of this type of problem is shown in Figs. 13 and 14.

The basic configuration used for this study was a blunt slab-delta-wing re-entry vehicle. The basic uncambered slab delta ($\phi=0^\circ$) had a leading-edge sweep of 75° and a nose and leading-edge radius of 4% of the body length. Variations of this basic shape were obtained by shearing the lower half of the vehicle downward to give a cambered lower surface. The amount of displacement at each body station was varied to give a flat forward lower surface, followed by a circular-arc surface contour to the aft end of the vehicle. The maximum thickness point was maintained at 55% of the body length. The cross-section of the top of the configuration was elliptical. The basic longitudinal stability characteristics for varying forward surface angle (ϕ) are shown in Fig. 14. These calculations were made by using modified Newtonian theory. The techniques described above have been used to study a very large number of shapes in a very short period of time.

The arbitrary-body analysis system has also been useful in studies of hypersonic wave-rider configurations such as those shown in Fig. 15. The

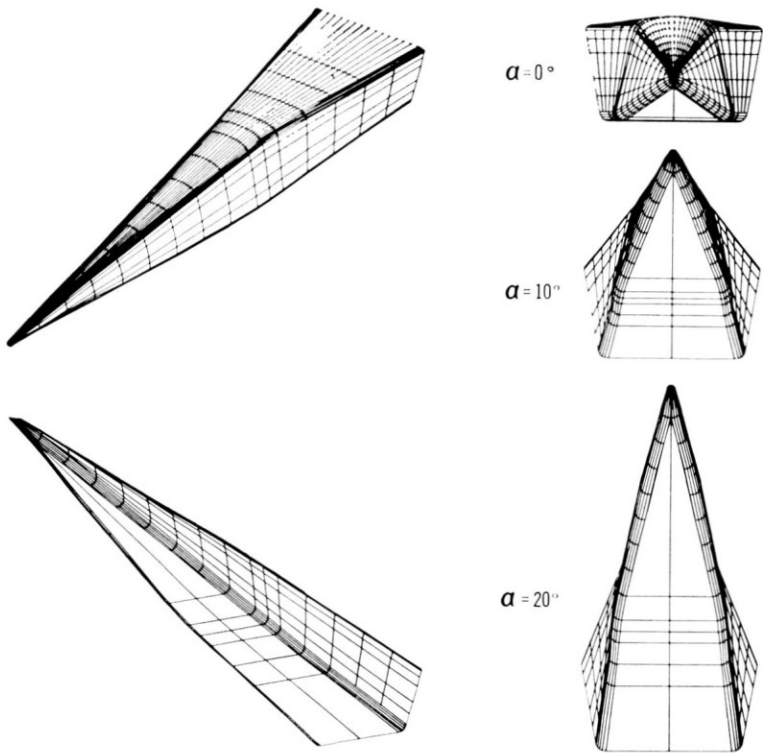


FIG. 11 — Geometric representation of lifting-body upper-stage vehicle

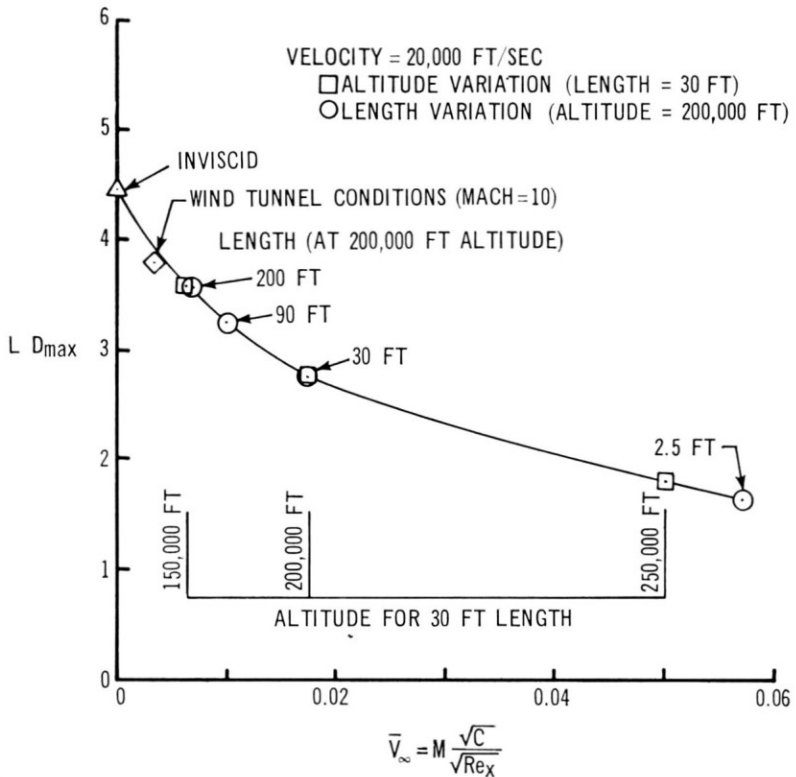


FIG. 12 — Lift-drag ratio characteristics of lifting-body upper-stage vehicle

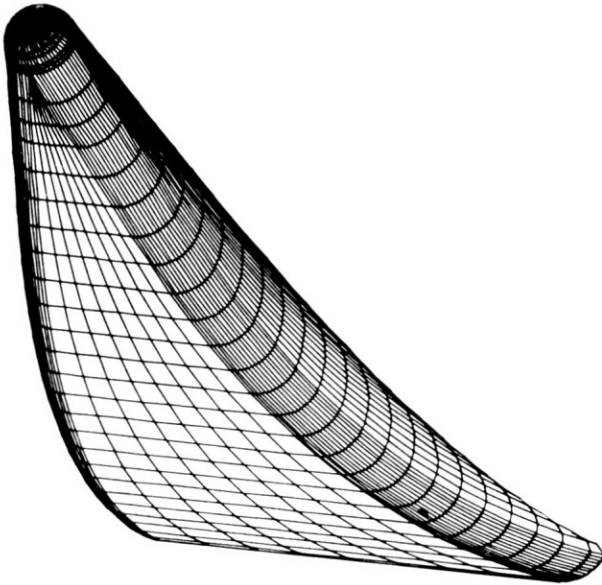


FIG. 13 — Geometric representation of lifting-body re-entry vehicle derived from slab-delta ($\phi = 15^\circ$)

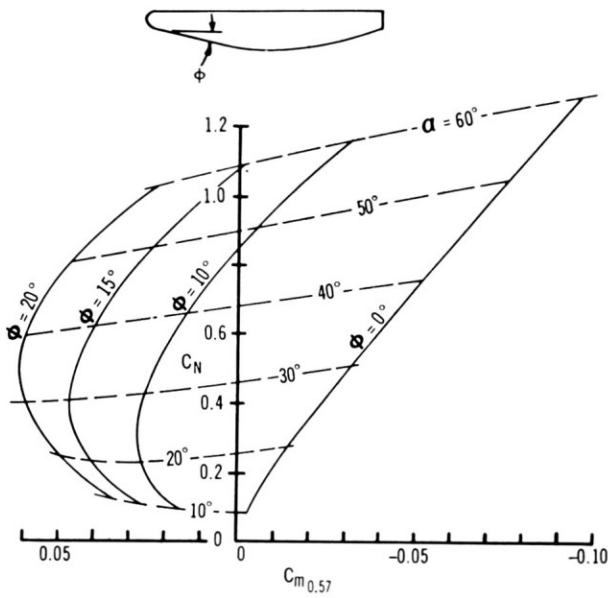


FIG. 14 — Effect of lower surface shape on longitudinal characteristics of lifting-body re-entry vehicle

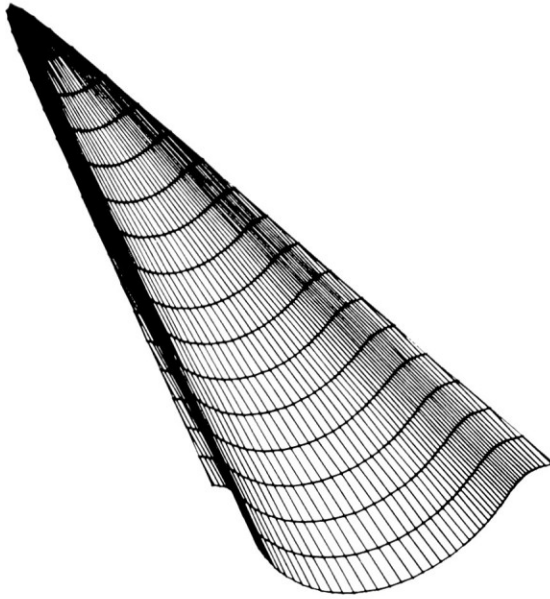


FIG. 15 — Geometric representation of wave-rider vehicle

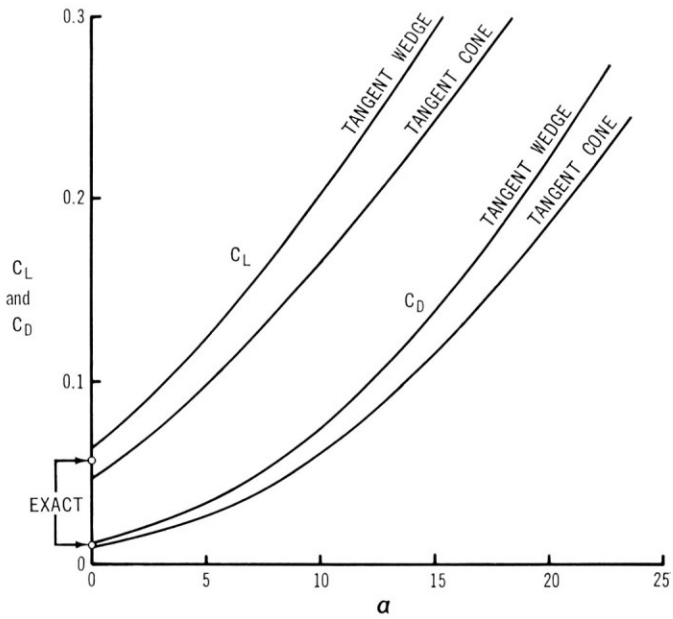


FIG. 16 — Lift and drag characteristics of wave-rider vehicle at off-design conditions

vehicle lower surface is defined so that it lies along the known stream-lines of the flow field generated by a nonlifting cone. This procedure, suggested by Jones^(2,3), gives a shape for which exact aerodynamic characteristics may be easily calculated (at the design conditions). The derivation of the vehicle shape and the calculation of exact aerodynamic characteristics at the design angle of attack and Mach number are accomplished by a separate computer program. The arbitrary-body analysis system is then used to produce drawings of the vehicle and to calculate approximate aerodynamic characteristics at off-design conditions where exact methods are not available. Typical results are shown in Fig. 16. This configuration was generated from the flow about a 9° semi-vertex-angle cone at a Mach number of 10. The top is flat and parallel to the free stream. The fore and aft lines on the picture in Fig. 15 are stream-lines on the body at the design condition.

6. CONCLUSIONS

The Hypersonic Arbitrary-Body Force Analysis System as described in this paper has proved to be a versatile and effective tool in performing aerodynamic analyses for complex vehicle configurations. With special techniques to simplify the input of geometric data, and with the use of computer-drawn pictures of these data to check for errors, it is possible to make excellent use of the distributed-plane-element method to describe complex vehicle shapes. The use of an arbitrary-body geometric description technique, together with a wide selection of theoretical and empirical force-calculation methods, has provided the capability of studying a large number of different vehicle shapes before resorting to the use of wind-tunnel tests. Excellent correlation has been obtained between calculated and experimental results, even for complex shapes previously difficult to analyze. The use of on-line graphical-display computer equipment provides an ideal means for facilitating the preparation of voluminous input geometric data and for obtaining a rapid analysis of computer output.

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DISCUSSION

David N. Reilly (Consultant, Ewing Technical Design, Inc., Philadelphia, Pa.): What would be the approximate elapsed time, in an automated system, between receiving a drawing of a typical hypersonic body and obtaining the final aerodynamic coefficients?

Mr. Gentry: The major time-consuming step in this analysis system is the preparation and validation of the geometric data. The time required depends upon the complexity of the vehicle shape. For a typical arbitrary shape such as a lifting-body re-entry vehicle this process may take as much as two to three days if the data are recorded manually, or as little as two hours if semi-automatic recording equipment is used. In a large company such as I work in, the total time that it takes from drawing to final answers is also largely a function of computer availability. However, with the availability of graphic display devices and computer equipment such as illustrated in the movie film the entire process would take from two to three hours.

Professor Dr. Engineer A. Varela Cid (Technical University-Centro Para Estudos Aeronauticos, Lisbon): I would like to congratulate you on your lecture because it was a very important demonstration of how the electronic computer can aid in the analysis of complex hypersonic shapes. Nevertheless, I would like to ask two questions about the re-entry vehicles that you used as illustrations in demonstrating your analysis methods. First, what is the Mach number and altitude relationship for these vehicles during a typical re-entry? Also, at altitudes near 30,000 feet there can exist turbulence and irregularities of the atmospheric temperature which can produce pressures and Mach numbers different than for a standard day. Atmospheric jet-streams may also be encountered where wind speeds may be as high as 400 miles per hour. These jet streams may be very large — up to 200 miles wide, 2.5 miles thick and 1000 miles long. Are these circumstances considered in your design studies of these re-entry vehicles?

Mr. Gentry: In a typical re-entry from a near-earth orbit the vehicle will fly through a very wide Mach number and altitude range. At the initial re-entry point at 300,000 feet the velocity will be about 26,000 feet per second (about Mach 30). At Mach 15 the nominal altitude would be about 200,000 feet. The vehicle would pass through 30,000 feet altitude at subsonic speeds. Variations in vehicle shape, and flight angle of attack and bank angle will give a wide variation in flight altitude at a given Mach number during the descent. As to your other question concerning non-standard days and wind

conditions — the effect of these phenomena is included in the design studies for these re-entry vehicles. At very high altitudes non-standard atmospheric conditions are considered both in the design requirements and in the detailed trajectory computation for the actual flight operations. The high wind and air turbulence effects that occur during the final terminal phase of the re-entry do not have a strong effect on the design of these vehicles. These vehicles usually have a low lift-curve slope at these conditions so are not as sensitive to air gusts as conventional aircraft are.